Use Bean Counting to teach Binomial Distribution in the Classroom

Owen Tang, The Hong Kong Polytechnic University, Hong Kong
Brian Sun, The Hong Kong Polytechnic University, Hong Kong

The IAFOR Conference for Higher Education Research – Hong Kong 2018
Official Conference Proceedings

Abstract
Since the 1980s, many academics have engaged in the research of statistics education. The underlying reason is that there was an increasing number of students taking introductory statistics courses, which stimulated the need to improve the teaching of statistics courses. Some researchers have suggested that teachers should focus more on concepts by designing more active learning activities. On the other hand, a substantial number of teachers have using the traditional lecture method. Some studies have found that an active learning technique has correlated with more positive attitudes or higher test scores but some studies showed a detrimental effect when using active learning methods in teaching business statistics. This paper reports the result of an educational experiment by dividing a class of 70 students (n=70) into two tutorial sessions (1-hour duration). One tutorial class was taught entirely with a lecture about the concept of binomial distribution. The other tutorial class was taught by using a minimal teacher-centered activity. At the beginning of the next tutorial class, an identical closed book exam of 20 minutes was conducted, and students' results on exams were analyzed. The result suggested that the activity session produced a better score both on conceptual questions and on application questions. However, one defect about this educational experiment is that the author did not control other factors that may affect the exam performance, such as the impact of previous GPA performance of the students in the two groups.

Keywords: teaching statistics, type of data, sample, binomial distribution

Introduction

The idea of using games and activities can be a better educational alternative in creating a fun learning environment in classroom settings. Educators and teachers have increasingly incorporated various games into their teaching curriculum. Although using games in classroom learning can be time consuming because they involve interactive communications and collaborations among students, they are very effective teaching tools for motivating student participation in the learning process. In McLester's 2005 article entitled "Game Plan", he investigated the use of games in U.S. major companies and the military, he found that "Nearly seventy percent of students learn best actively and visually" (McLester, 2005). Quinn and Iverson indicated that game activities can enhance students' learning experience by making them the active participants. In short, game activities can help the students by placing them "at the centre of the learning experience" (as cited in Pannesse & Carlesi, 2007).

When we used the bean counting activity in the classroom, we found that it can serve as an effective tool to help the students in understanding the statistical concept of binomial distribution which delivered in using the traditional lecturing method. We have noticed that most students voiced their enjoyment in using hands-on activities in the learning process. Although we may not be certain that hands-on activities could replace the traditional lecture format, it seems that they were very good supplements in understanding the lecturing materials.

By assessing the results of our bean counting activities, we found that many students did receive the positive learning benefits because the hands-on activities offered a chance of active participations in the learning process.

Binomial Distribution

Two of the most widely used discrete probability distributions are the binomial and Poisson. In analyzing statistical data which can be counted rather than measured, statisticians frequently use the concept of binomial distribution. The binomial distribution is now widely used to analyze data in almost every field of human inquiry.

For example, in 1936 the British statistician Ronald Fisher used the binomial distribution to work on the famous experiments on pea genetics reported by Gregor Mendel in 1866. Fisher observed that Mendel's laws of inheritance would dictate that the number of yellow peas in one of Mendel's experiments would have a binomial distribution with n = 8,023 and p = $^3/_4$, for an average of $np \cong$ 6,017 yellow peas. Fisher found remarkable agreement between this number and Mendel's data, which showed 6,022 yellow peas out of 8,023. By using the binomial distribution, Fisher found that all seven results in Mendel's pea experiments were extremely close to the expected values.

It applies to any fixed number (n) of repetitions of an independent process that produces a certain outcome with the same probability (p) on each repetition. For example, it can provide a distribution for the probability of obtaining 10 sixes in 50 rolls of a die. Swiss mathematician Jakob Bernoulli determined that the probability of k such outcomes in n repetitions is equal to the kth term (where k starts with 0) in the expansion of the binomial expression $(p + q)^n$, where $q = 1 - p$. In the example of the die, the probability of turning up any number on each roll is 1 out of 6 (the number of faces on the die). The probability of turning up 10 sixes in 50 rolls, then, is equal to the 10th term (starting with the 0th term) in the expansion of $(5/6 + 1/6)^{50}$, or 0.115586.

Teachers who use traditional lecturing method can show an explicit formula for the *kth* term of a binomial expansion by a binomial theorem $f(x) = \binom{n}{x} p^x (1-p)^{(n-x)}$. However, in order to help students in understanding the binomial distribution, show a visual presentation of the data would be a good way to point out facts which might otherwise be overlooked.

In our designed game, we adopt a special type of graph paper which was designed by Frederick Mosteller (Harvard University) and John W. Tukey (Princeton University). Mosteller and Tukey's visual presentation method can be traced back to R. A. Fisher's observation that:

$$cos^2 \phi_i = \frac{n_i}{n}$$

transformed the multinomial distribution with observed number $n_1, n_2, \cdot \cdot \cdot, n_k$ into direction angles $\phi_1, \phi_2, \cdot \cdot \cdot, \phi_k$ which were nearly normally distributed with individual variances nearly $1/4n$ (when the angles are measured in radians). Thus the point at a distance $\sqrt{n}$ from the origin and in the direction given by $\phi_1, \phi_2, \cdot \cdot \cdot, \phi_k$ is distributed on the $(k-1)$ dimensional sphere nearly normally, and with variance nearly ¼ independent of n and the true fractions $p_1, p_2, \cdot \cdot \cdot, p_k$ of the different classes in the population. The rectangular coordinates of this point are $\sqrt{n_1}$, $\sqrt{n_2}, \cdot \cdot \cdot, \sqrt{n_k}$.

Mosteller and Tukey's graph paper employed R.A. Fisher's inverse sine transformation for proportions. The transformation itself is designed to adjust binomially distributed data so that the variance will not depend on the true value of the proportion *p*, but only on the sample size *n*. In addition, binomial data so transformed more closely approximate normality than the raw data (Mosteller and Tukey, 1949). By using the special designed graph paper, plotting binomial data in rectangular co-ordinates, using a square-root scale for the number observed in each category would makes the angular transformation $p = cos^2 \phi$ or $p = sin^2 \phi$ easily available at the same time. With such paper, most tests of counted data can be made quickly, easily and with what is usually adequate accuracy. In Mosteller and Tukey's article, they gave 22 examples to demonstrate the use of such plotting method.

Good Educational Game

What are the factors that determine a good "educational game"?
Some education games are by nature competitive; while other games just simply allow students to work together as a team to solve a general problem. Prior researches indicated that by supplementing traditional lectures with active learning activities in the classroom, as summed up by Franklin, Peat & Lewis (2003), "games foster group cooperation and typically create a high level of student involvement that makes them useful tools for effective teaching".

Okan (2003) questioned whether or not learning should always have to be “fun.” Some educators argue that “meaningful learning may sometimes be difficult and requires cognitive and emotional effort, especially in considering the fact that post-secondary education is not usually a fun undertaking”. Other educators view that the mere act of problem solving by itself is full of fun. In MacKenty’s 2006 paper entitled “*All Play and No Work*”, it observed that “it is the act of problem solving that makes games so engaging”.

Then the next step of investigation is whether the risk of failure in participating in a competitive learning game would destroy the fun part of the learning process. Despite the feeling of failure, Schaller (2006) noted that by repeating the game activities, it may encourage students to work on those high level thinking skills, such as “experimentation, hypothesis testing and synthesis”. The positive aspect can be strengthened by using non-competitive games, in Tom Schrand’s 2008 published article in *Collegiate Teaching*, he discussed how interactive multimedia activities can help students to work together as a team in grouping relevant information and facts into proper categories.

Harris (2009) found that if an educator can choose a well-designed game in the classroom, regardless of whether the game is competitive or non-competitive, it helped the students to build their problem solving skills while having fun simultaneously. His paper also investigated the best way to integrate the game activity into the teaching curriculum (Harris, 2006, p.26). Van De Bogart (2009) conducted a research about how the personal beliefs on teaching pedagogy of instructors would affect their choices of educational activities in classroom.

Teachers’ intent

Teachers’ intent is one of the key factors affecting the values of educational games. Audrey Amrein-Beardsley (2009) found that many teachers were simply “teaching to the test” with games. Their intent was merely helping the students to “become experts at answering the test questions without entirely understanding the concepts justifying their answers”. She argued that such underlying purpose may negatively affect the “inquiry-based, higher order, problem-solving activities” that we valued the most (Beardsley 2009).

The very purpose of a good education games is active learning, and active learning can be defined in many ways. One way is to define it as “an effort to make learning authentic” (Van De Bogart, 2009). In addition, active learning can be referred to teaching techniques that enable students to engage in something rather than merely listening to a lecture, such as “discovering, processing and applying new information”.

Finally, one last concern regarding educational games comes from a recent case study that focused on teachers adopting educational computer games. Kebritchi (2010) poses the concern that games are becoming such innovative learning tools that teachers may conclude that they don’t need to lecture, and instead they intent to “rely on the game and use it as a teaching replacement and not as a supplement” (p. 263). It is important to remember that games are supplement teaching tools and teachers ultimately need to be actively involved for them to be truly effective.

Pre-game preparations

Rotter (2004) investigated the aspects of pre-game preparations. One way is by encouraging the "student to predict questions that will be asked on the test" and then providing the teams whose questions are chosen with bonus points on the game. This kind of pre-game preparations will motivate the students to study and prepare outside of the classroom, and such activities are very positive for additional reinforcement.

To address the issue of lagging student participation when one student is answering the question, teachers can "ask all student to bring their prepared notes to class on the day of the game" and then "instruct all pupils to add or highlight the answers to questions as part of the game" (Rotter, 2004). Although Rotter's 2004 paper did not address the issue of how to incorporate the pre-game preparations into scoring, it would be beneficial to give additional points for students as their incentives.

Assessment

There are many computer based games that can provide active learning opportunities and can reinforce topics learned in the classroom. One issue the teachers need to consider is the nature of students' interactions with the computers. Will the students work independently and individually compete against the computer? And in what way such individual efforts can promote cooperative learning? Teachers may divide the game into two stages. The first stage consists of competing with the computer individually for "skill exercises". The second stage involves students compete against other students in a team. Such arrangement tend to help the learning process because it provides "both group rewards and individual accountability"

In their case study, Ke and Grabowski (2007) compared pre-test and post-test results of three groups of students. The first group competed using the TGT format (cooperative gameplay.) The second group of students worked independently as they competed individually with the computer and their scores were posted weekly to compare their results with their peers (competitive gameplay.) The third group participated in paper/pencil review sessions and did not play the games at all (control.) Results showed that there "was no significance for maths performance between cooperative game playing and competitive game playing but both performed significantly higher than the control group".

Another teaching methodology that incorporates active and cooperative learning pedagogies is the knowledge net framework (Williamson, Lee, Butler, Ndahi, 2004). The researchers selected a group of fifth grade students using the rules of baseball. Students were divided into two teams where they were allowed to choose the teams and questions were provided beforehand. For the game, each student takes a turn "at bat" to score a "hit" by answering the question correctly. Players advance one base at a time. As in baseball, if a student gets a question wrong, he is "out". After three outs, the other team takes a turn "batting". The researchers found that the student achievement in science at the test school showed dramatic gains. In addition, the game fosters self-categorization that motivate the students to learn science in meaningful ways.

Conclusion

This paper conducts a brief literature review, which gives the readers a greater understanding of the benefits and constraints of using games in the classroom. Games conducted in classroom would require pre-game preparations conducted outside of the classroom through studying the process and review questions beforehand. By participating in a game, it provides rewards to the participants yet still holds students accountable.

Certainly, incorporating games in the learning process would require significantly more class time to prepare and conduct when compared to the traditional lecturing method. Based on our studies, we would predict that the games would motivate the students to become active learners in the classroom setting. And it would generate evidences for our assessments. We can compare students' performance in plotting the binomial distribution in the graph paper with their performances in their final examination.

Appendix

Make a frequency table and work out the relative frequency of getting red beans and green beans.

No. of red beans	frequency	Relative frequency = $\frac{frequency}{total\ number\ of\ beans\ in\ each\ trial}$
0		
1		
2		
3		
4		
. . .		

Table 1: Frequency Table.

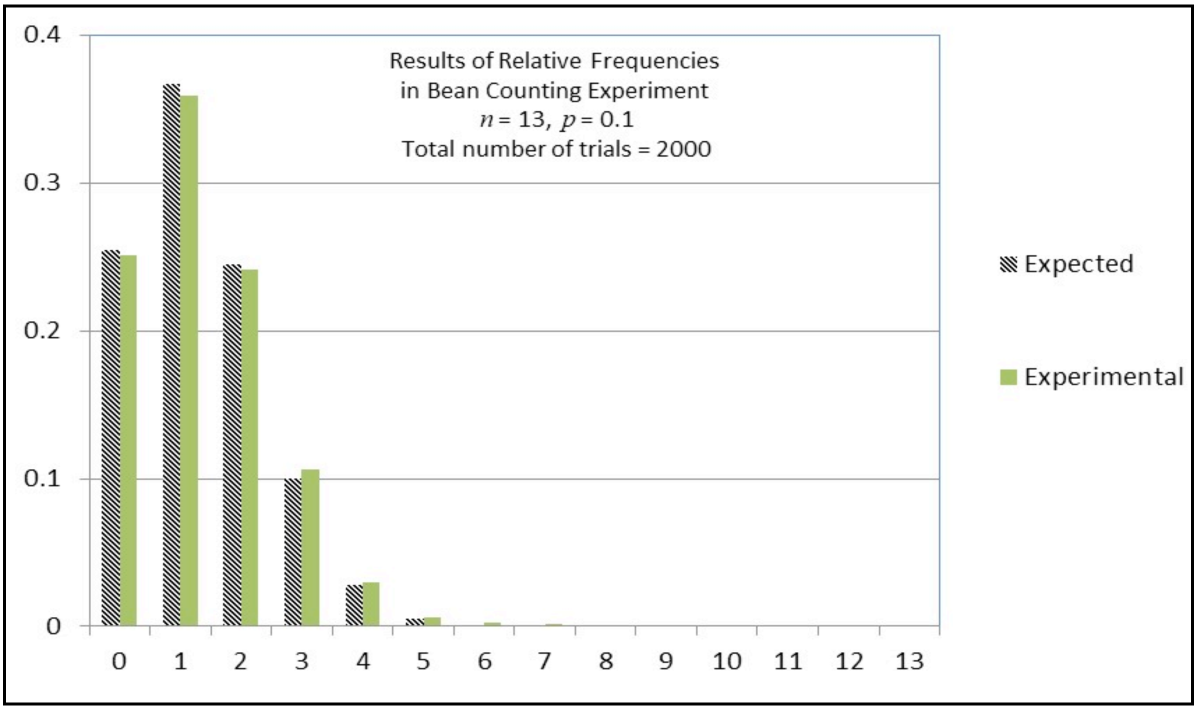

Figure 1: Results of Relative Frequencies in the bean counting activity.

Acknowledgements

This study was supported in part by The Hong Kong Polytechnic University under the Learning and Teaching Enhancement Grants Scheme 2017/18 (account code 1.44.xx.8AD3).

References

Amrein-Beardsley, A. (2009). “This is Jeopardy!”. *Education Digest, 74*, 14-18.

Franklin, S., Peat, M., & Lewis, A. (2003). Nontraditional interventions to stimulate discussion: the use of games and puzzles. *Journal of Biological Education, 37*, 79-84.

Harris, C. (2009). Meet the New School Board: Board games are back – and they are exactly what your curriculum needs. *School Library Journal, 55,* 24-26.

Ke, F., & Grabowski, B. (2007). Gameplay for maths learning: cooperative or not?. *British Journal of Educational Technology, 37,* 249-259.

Kebritchi, M. (2010). Factors affecting teachers’ adoption of educational computer games: A case study. *British Journal of Educational Technology, 41,* 256-270.

MacKenty, B. (2006). All Play and No Work. *School Library Journal*, *52,* 46-48.

McLester, S. (2005). Game Plan. *Technology and Learning, 26*(3), 18-20, 22, 24, 26.

Mosteller, F. & Tukey, J. (1949) The Uses and Usefulness of Binomial Probability Paper. *Journal of the American Statistical Association*, *44*(246), 174-212.

Okan, Z. (2003). Edutainment: is learning at risk? *British Journal of Educational Technology, 34,* 255-264.

Pannese, L. & Carlesi, M. (2007). Games and learning come together to maximize effectiveness: The challenge of bridging the gap. *British Journal of Educational Technology, 38*(3), 438-454.

Rotter, K. (2004). Modifying “Jeopardy!” Games to Benefit All Students, *Teaching Exceptional Children, 36,* 58-62.

Schaller, D. (2006). What Makes A Learning Game? Retrieved from http://www.eduweb.com/schaller-games.pdf

Schrand, T. (2008). Tapping into Active Intelligences with Interactive Multimedia: A Low threshold Classroom Approach. *Collegiate Teaching, 56,* 78-84.

Williamson, K. M., Land, L., Butler, B., & Ndahi, H. B. (2004). A structured framework for using games to teach mathematics and science in K-12 classrooms. *The Technology Teacher, 64,* 15-18.

Van De Bogart, W. (2009). Developing a Pedagogy for Active Learning (PAL) Part I. Retrieved from http://www.earthportals.com/Portal_Messenger/ActiveLearning.html

Contact email: brian.pw.sun@polyu.edu.hk